On possibility of turbulence wave number spectra reconstruction using radial correlation reflectometry in Tore Supra and FT-2 tokamaks

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10th International Reflectometry Workshop, May, 4th - 6th 2011
Outline

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- Numerical modeling of the micro turbulence radial wave number spectrum and CCF reconstruction in different cases:
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  2. Complicated plasma density profile (Tore Supra, France)
  3. Inhomogeneous spectrum reconstruction (Tore Supra, France)
- Conclusions and further plans
Introduction
### Fluctuation reflectometry

The reflected signal phase: \[ \Phi(\omega) = \Phi(\omega) + \delta\Phi \]

Density profile reconstruction:

\[
\Phi(\omega) = \int_{x_0}^{x_c} k_i(\omega, x) \, dx = \frac{\omega}{c} \int_{x_0}^{x_c} \sqrt{1 - \frac{n(x)}{n_c(\omega)}} \, dx
\]

Fluctuations:

\[
\delta\Phi(\omega) = \frac{\omega}{c} \int_{x_0}^{x_c} \frac{\delta n(x)}{n_c(\omega)} \left( \sqrt{1 - \frac{n(x)}{n_c(\omega)}} \right) \, dx
\]

**Advantages of the method:** the possibility to associate the location of transmitting antenna and the receiver; reasonable measurements localization.

**The main disadvantage:** not enough fluctuation wave number resolution.
Radial correlation reflectometry (RCR)

\[ \Delta L = L_a - L_b \quad \Delta \omega = \frac{\omega_b - \omega_a}{L} \]
- distance between cut offs

\[ \Delta \omega = \omega_a - \omega_b \]
- \( \omega_a \) - reference frequency
- \( \omega_b \) - probing frequency

Simultaneously different frequencies for probing are used, the cross-correlation function (CCF) of probing signals takes a form:

\[
CCF = \left\langle A_s(\omega_a)A_s^*(\omega_b) \right\rangle
\]

\( A_s(\omega_a) \) - received signal
**RCR data interpretation problems**

Large difference between RCR CCF and turbulence CCF in 1D Born approximation and full-wave computations:

\[ L - \text{measured, } l_c - \text{real turbulence correlation length} \]

RCR data interpretation problems

Large difference between RCR CCF and turbulence CCF in 2D Born approximation computations and theory:

In agreement with 2D slab plasma theory prediction by Gusakov E.Z., Yakovlev B.O. PPCF 44 (2002) 2525-2537

The turbulence wave number spectrum reconstruction background
Linear theory of RCR

1D model Helmholtz equation for O-mode:

\[
\left\{ \frac{d^2}{dx^2} + \frac{\omega^2}{c^2} - \frac{4\pi e^2}{m_e c^2} \left[ n(x) + \delta n(x) \right] \right\} E_z(x, \omega) = 0
\]

\[
n(x) = n_e \frac{x}{L}
\]

- linear density profile;

\[
\delta n(x) = \frac{1}{2\pi} \int \delta n_k e^{-ikx} dk
\]

- homogeneous density perturbations, \( k \) is a radial wave number;

\[
\omega
\]

- probing frequency;

\[
E_z
\]

- total field of the probing wave;

\[
e - \text{electron charge, } m_e - \text{electron mass and } c - \text{velocity of light.}
\]
Linear theory of RCR

in linear (Born) approximation:

\[ \frac{\delta n(x)}{n_c} \ll 1 \]

\[ A_s(\omega) = \frac{i\omega \sqrt{S_i}}{16\pi} \int_0^\infty \frac{\delta n(x)}{n_c} E_0^2(x, \omega) dx \]

Reciprocity theorem [6]

- incident wave energy flux density;
- unperturbed electric field.

\[ CCF = \left\langle A_s(\omega_a)A_s^*(\omega_b) \right\rangle \]

Turbulence spectrum reconstruction

The normalized CCF for linear density profile

\[
CCF(L) \propto \int_{-\infty}^{+\infty} \frac{d\kappa}{|\kappa|} \tilde{n}_\kappa^2 e^{i\kappa \Delta L} \text{erf} \left( \sqrt{i\kappa L_0} \right) \text{erf}^* \left( \sqrt{i\kappa L} \right)
\]

\[\Delta L = L_0 - L\]

cut-off separation,

\[
\text{erf} \left( s \right) = \int_{0}^{s} e^{-\zeta^2} d\zeta
\]

\[\tilde{n}_\kappa^2\]

turbulence radial wave number spectrum

correlation function:

\[
2\pi \langle \delta n(x') \delta n(x'') \rangle = \delta n^2 \int_{-\infty}^{\infty} \tilde{n}_\kappa^2 \exp \left[ i\kappa (x' - x'') \right] d\kappa
\]

turbulence spectrum in terms of CCF

\[
n_\kappa^2 \propto \left\langle \frac{2}{\sqrt{\pi}} e^{\frac{i}{2}(1+\text{sign}(\kappa))} \right\rangle \int_{-\infty}^{+\infty} \frac{d\kappa}{\text{erf}^* \sqrt{i\kappa L_0}} \int CCF(\Delta L) e^{i\kappa \Delta L} d\Delta L
\]
Numerical procedures

The scheme of numerical computations:

\[ \delta n(x) = \frac{1}{2\pi} \int \delta n_\kappa e^{-i\kappa x} d\kappa \]

\[ A_s(\omega) = \frac{i\omega \sqrt{S_i}}{16\pi} \int_0^\infty \frac{\delta n(x)}{n_c} E_{z0}^2(x, \omega) dx \]

\[ CCF(\Delta L) = \frac{\left\langle \left( A_s(\omega_0) - \langle A_s(\omega_0) \rangle \right) \left( A_s(\omega_1) - \langle A_s(\omega_1) \rangle \right)^* \right\rangle}{\sqrt{\left\langle \left( A_s(\omega_0) - \langle A_s(\omega_0) \rangle \right)^2 \right\rangle \left\langle \left( A_s(\omega_1) - \langle A_s(\omega_1) \rangle \right)^2 \right\rangle}} \]

\[ n_\kappa^2 \propto \frac{2}{\sqrt{\pi}} e^{\frac{i\pi}{2}(1+\text{sign}(\kappa))} \frac{\left| \kappa \right|}{\text{erf}^* \sqrt{i\kappa L_0}} \int_{-\infty}^{+\infty} CCF(\Delta L)e^{i\kappa \Delta L} d\Delta L \]
Numerical modeling of the micro turbulence radial wave number spectrum and CCF reconstruction from the RCR data
FT-2 tokamak

Plasma density profile on FT-2 tokamak, Saint-Petersburg, Russia

\[ n(x) = n_c \frac{x}{L_0} \]

Approximately linear plasma density profile [7]:

\[ n_c = 1 \div 6 \cdot 10^{13} \text{ cm}^{-3} \]

\[ a = 0.08m \]
\[ R = 0.55m \]

minor radius;
major radius;
plasma density;
probing frequency range for O-mode.

\[ 26 \text{GHz} < f < 36 \text{GHz} \]

discharges # 56 - 72

\[ N_e, \text{ cm}^{-3} \]
\[ R_{ms}, \text{ cm} \]

FT-2 tokamak: signal & turbulence CCF

Calculation parameters:

\[ f_0 = 31.7 \text{GHz} \]  
reference frequency;

\[ 24.5 \text{GHz} < f < 37.5 \text{GHz} \]  
probing frequency range for O-mode;

\[ L_0 = 0.05m \]  
reference cut off;

\[ 0.03 \text{m} < L < 0.07 \text{m} \]  
probing interval.

Plasma turbulence parameters:

\[ \tilde{n}_k^2 = \sqrt{\pi} l_c e^{-l_c^2 k^2 / 4} \]  
Gaussian radial wave number spectrum;

\[ l_c = 0.02m \]  
correlation length.

Signal CCF compared to turbulence CCF

The averaging is performed over ensemble of 500 random phase samples.
The method is applicable for higher densities (4.0 – 8.0 E19 m\(^{-3}\)) subject to utilizing higher frequencies up to 80GHz and probing range \(\Delta L / \lambda \approx 6...15\).

\(F T-2: \) reconstruction

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**Graph (a)**: Reconstructed spectrum compared to Gaussian shown in relative units

- Re (spectrum)
- Im (spectrum)
- Gauss

Plasma top density: \(n_t = 2 \cdot 10^{19} m^{-3}\)

**Graph (b)**: Reconstructed turbulence CCF compared to the input Gaussian

\[\Delta L = 0.04m\]
\[\lambda = 0.0095m\]

\[\Delta L / \lambda = 4.2\]
**FT-2: reconstruction**

(a) Reconstructed spectrum compared to Gaussian shown in relative units

- Plasma top density: \( n_t = 4 \cdot 10^{19} m^{-3} \)
- Correlation length: \( l_c = 0.005 m \)

(b) Reconstructed turbulence CCF compared to the input Gaussian

\[
\Delta L = 0.06 m \\
\lambda = 0.0075 m
\]

\[\Delta L / \lambda = 8\]

Probing frequency range for O-mode: \(21.1 GHz < f < 52.7 GHz\)
Tore Supra tokamak

- $a = 0.7m$ minor radius;
- $R = 2.25m$ major radius;
- $n_c = 2 \div 7 \cdot 10^{19} \text{ cm}^{-3}$ plasma density;
- $26\text{GHz} < f < 160\text{GHz}$ probing frequency range for $O$ and $X$ modes.

**Tore Supra: standard density profile**

Synthetic Tore Supra like plasma density profile:

\[
n(x) = n_c \cdot 0.5 \left(1 + \tanh(\alpha (x - x_a))\right) \cdot \left(1 - \left(x - x_b\right)^2 / a^2\right)
\]

\[
\alpha = 80 m^{-1}, \ x_a = 0.05 m, \ x_b = 0.8 m, \ a^2 = 0.75
\]

\[
0.26 m < L < 0.46 m \quad f_0 = 47.8 GHz \quad L_0 = 0.036 m
\]

Radial wave number spectrum [9]:

\[
\delta n_k^2 = \begin{cases} 
1, & \left|\kappa\right| < 1 cm^{-1} \\
\kappa^{-3}, & 1 cm^{-1} < \left|\kappa\right| < 6 cm^{-1} \\
\kappa^{-6}, & \left|\kappa\right| > 6 cm^{-1}
\end{cases}
\]

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(a) Reconstructed spectrum compared to input shown in relative units
Probing step: \( h = 0.001 \) m
Number of frequencies used: \( N = 800 \)
Number of averaging modes: \( N_m = 500 \)

(b) Reconstructed turbulence CCF compared to the input one
plasma top density:
\[ n_t = 3.9 \cdot 10^{19} m^{-3} \]
Tore Supra: reconstruction

Radial wave number spectrum [10]:

\[ \delta n_k^2 = \begin{cases} \frac{k^{-3}}{(1+k^2)^2}, & \kappa \rho_s \geq 0.1 \\ 1, & \kappa \rho_s < 0.1 \end{cases} \]
Tore Supra: reconstruction

(a) Reconstructed spectrum compared to input shown in relative units
Probing step: $h=0.001m$
Number of frequencies used: $N=800$
Number of averaging modes: $Nm=500$

(b) Reconstructed turbulence CCF compared to the input one
plasma top density:

$$n_t = 8.5 \cdot 10^{19} m^{-3}$$
ITER like density profile

Synthetic plasma density profile:

\[ n(x) = n_c \cdot 0.5 \left( 1 + \text{th} \left( \alpha (x - x_a) \right) \right) \cdot \left( 1 - \frac{(x - x_b)^2}{a^2} \right) \]

\[ \alpha = 100 \text{m}^{-1}, \quad x_a = 0.045 \text{m}, \quad x_b = 0.8 \text{m}, \quad a^2 = 2.55 \]

Probing intervals:

1. \(0.01 \text{m} < L < 0.128 \text{m}\)
   \[ f_0 = 73.3 \text{GHz} \quad L_0 = 0.069 \text{m} \]

2. \(0.2 \text{m} < L < 0.4 \text{m}\)
   \[ f_0 = 78.6 \text{GHz} \quad L_0 = 0.3 \text{m} \]


(a) Turbulence CCF reconstruction, asymmetric plasma density profile in the probing interval. $L_0 = 0.069m$

(b) Turbulence CCF reconstruction, parabolic plasma density profile, symmetric case. $L_0 = 0.3m$
Inhomogeneous turbulence

(a) CCF, reference cut off $L_0 = 0.4m$

Turbulence spectrum $\tilde{n}_\kappa^2 = \sqrt{\pi l_c} e^{-l_c^2 \kappa^2 / 4}$

$l_c = \begin{cases} 
0.01m, & 0.37m < L_1 < 0.43m \\
0.02m, & \text{otherwise}
\end{cases}$

the probing interval $0.37m < L_1 < 0.43m$

(b) turbulence CCF reconstruction

Turbulence parameters reconstruction inside the transport barrier $l_c = 0.01m$

Linear plasma density profile:

$n(x) = n_c x/L$
The transport barrier signature in CCF

(a) CCF, reference cut off \( L_0 = 0.5m \)  (b) turbulence CCF reconstruction \( l_c = 0.02m \)

Turbulence parameters reconstruction outside the transport barrier shows the sensitivity of the method to local turbulence characteristics.

Transport barriers detection:
Signal CCF - signature of a thin layer (the CCF inside barrier is flat) - the reference frequency cut off is put inside transport barrier - measurements of the turbulence characteristics inside barrier
Conclusions and further plans
Conclusions

- The simple linear 1D theory both for linear and arbitrary plasma density profiles cases showing the possibility of turbulence radial wave number spectrum reconstruction was developed;

- Numerical modelling of the micro turbulence radial wave number spectrum and CCF reconstruction for small FT-2 tokamak represents the close to theory case of almost linear density profile;

- The case of complex plasma density profile with a steep gradient close to the plasma edge (Tore Supra) demonstrates the possibilities of developed procedure: it is applicable for modelling in conditions close to experiments;
Further plans

- Modelling of plasma turbulence characteristics reconstruction using 2D Born approximation and full wave code;

- Based on the developed method, reconstruction of the turbulence parameters from the experimental RCR data obtained at FT-2 (Russia), Tore Supra (France) and JET (UK) tokamaks.
Thank you for your attention!
Appendix: Noise influence

The way to introduce noise:

\[ N_s = A_s + nA_s e^{i\varphi(\omega_j)} \]

- \( N_s \) - total signal
- \( A_s \) - simulated signal
- \( n \) - noise level
- \( \varphi(\omega_j) \) - accidental uncorrelated phase

\[
\overline{CCF} = \frac{\langle N_s(\omega_a)N_s^*(\omega_b) \rangle}{\sqrt{\langle N_s^2(\omega_a) \rangle \langle N_s^2(\omega_b) \rangle}}
\]
Appendix: white noise reconstruction

Noise level:

\[ n=0.1 \quad n=1.0 \]
Appendix: conditions relevant to experiment

CCF including noise 100%, range of probing 3.2lc, probing step 0.32lc
Appendix: limitations of linear case

Limitations due to multiple small angle scattering in large machines and at high turbulence level

\[
\sigma^2 \approx \frac{\omega^2 l_c x_c}{c^2} \left( \frac{\delta n}{n_c} \right)^2 \ln \frac{x_c}{l_c} \geq 1
\]

the linear analysis is not valid